Field fluctuations near a conducting plate and Casimir-Polder forces in the presence of boundary conditions

S. Spagnolo, R. Passante and L. Rizzuto

CNISM and Dipartimento di Scienze Fisiche ed Astronomiche,

Università degli Studi di Palermo,

*Via Archirafi 36, I-90123 Palermo, Italy**

Abstract

We consider vacuum fluctuations of the quantum electromagnetic field in the presence of an infinite and perfectly conducting plate. We evaluate how the change of vacuum fluctuations due to the plate modifies the Casimir-Polder potential between two atoms placed near the plate. We use two different methods to evaluate the Casimir-Polder potential in the presence of the plate. They also give new insights on the role of boundary conditions in the Casimir-Polder interatomic potential, as well as indications for possible generalizations to more complicated boundary conditions.

PACS numbers: 12.20.Ds, 42.50.Ct

*Electronic address: roberto.passante@fisica.unipa.it

I. INTRODUCTION

Electromagnetic vacuum fluctuations have remarkable effects on atomic or molecular systems, such as energy level shifts and spontaneous decay. Another important consequence of the existence of field fluctuations in the vacuum are Casimir-Polder forces, which are long-range interactions between neutral atoms or molecules [1]. Casimir-Polder forces are also related to the Casimir effect, that recently has been measured with remarkable precision [2, 3]. The Casimir-van der Waals interaction between an atom and a dielectric wall has also been measured [4, 5].

It is well known that vacuum fluctuations change when boundary conditions are present, due to the change of the field modes [6, 7, 8]. The aim of this paper is to study how the physical properties of the vacuum state, in particular the fluctuations of the electric field, are modified by the presence of an infinite perfectly conducting plate, and how these changes reflect upon the Casimir-Polder interaction between two atoms placed near the conducting plate. The methods used in this paper give also useful indications on possible extensions to more complicated cases, such as different geometries of the boundary conditions or dielectric rather than metallic objects. The results obtained in this paper also provide new physical models which may be helpful for understanding the origin and peculiarities of Casimir-Polder forces with boundary conditions, such as the suggested enhancement of these forces between atoms inside a dielectric slab [9].

II. FIELD FLUCTUATIONS NEAR A PERFECTLY CONDUCTING PLATE

In the multipolar coupling scheme, the Hamiltonian describing two atoms A and B interacting with the radiation field in the dipole approximation is

$$\begin{aligned} H &= H_F + H_A + H_B + H_{AF} + H_{BF} \\ &= \sum_{\mathbf{k}j} \hbar\omega_k a^\dagger_{\mathbf{k}j} a_{\mathbf{k}j} + H_A + H_B - \boldsymbol{\mu}_A \cdot \mathbf{D}(\mathbf{r}_A) - \boldsymbol{\mu}_B \cdot \mathbf{D}(\mathbf{r}_B) \end{aligned} \tag{1}$$

where

$$\mathbf{D}(r) = \sum_{\mathbf{k}j} \mathbf{D}(\mathbf{k}j, \mathbf{r}) = i \sum_{\mathbf{k}j} \sqrt{\frac{2\pi\hbar\omega_k}{V}} \mathbf{f}(\mathbf{k}j; \mathbf{r}) \left(a_{\mathbf{k}j} - a^\dagger_{\mathbf{k}j} \right) \tag{2}$$

is the transverse displacement field operator, that in this coupling scheme is the momentum conjugate to the vector potential, and $\mathbf{f}(\mathbf{k}j; \mathbf{r})$ are the appropriate mode functions taking

into account the boundary conditions for the field operators (j is a polarization index) [10]. The mode functions, which we take as real functions, satisfy the normalization condition

$$\frac{1}{V}\int d^3r \mathbf{f}(\mathbf{k}j;\mathbf{r})\mathbf{f}(\mathbf{k}'j';\mathbf{r}) = \delta_{\mathbf{k}\mathbf{k}'}\delta_{jj'} \tag{3}$$

Compared to the free-space case, the presence of the conducting plate yields a change of vacuum fluctuations, and consequently changes of radiative processes of atoms placed near the plate (e.g. level shifts and decay rates) [6]; another consequence is a Casimir-like potential energy between an atom and the plate [11]. Due to the boundary conditions, the energy density of vacuum fluctuations also changes [8, 12], and this is of course related to the atom-wall Casimir potential. We shall show that another direct consequence is a change in the Casimir-Polder intermolecular potential between two atoms compared to the free-space case.

Vacuum fluctuations have equal-time spatial field correlations, whose properties are related to the Casimir-Polder potential between two neutral atoms [13]. Field correlations are also affected by the boundary conditions, as in the present case of a perfectly conducting plate. This yields a modification of the Casimir-Polder intermolecular potential between two atoms, as we shall discuss in detail in the next Section. In the vacuum state $\mid \{0_{\mathbf{k}j}\}\rangle$, the equal-time spatial correlation of the displacement field at points $\mathbf{r}_A$ and $\mathbf{r}_B$ is given by

$$\langle\{0_{\mathbf{k}j}\}\mid D_\ell(\mathbf{k}j;\mathbf{r}_A)D_m(\mathbf{k}j;\mathbf{r}_B)\mid\{0_{\mathbf{k}j}\}\rangle = \frac{2\pi\hbar ck}{V}\sum_{\mathbf{k}j} f_\ell(\mathbf{k}j;\mathbf{r}_A)f_m(\mathbf{k}j;\mathbf{r}_B) \tag{4}$$

In the case of an infinite metallic plate, the sum over polarizations and the angular part of the integration over $\mathbf{k}$ can be easily performed, obtaining [10]

$$\frac{1}{4\pi}\int d\Omega_k \sum_j f_\ell(\mathbf{k}j;\mathbf{r}_A)f_m(\mathbf{k}j;\mathbf{r}_B) = \tau_{\ell m}(kR) - \sigma_{\ell n}\tau_{nm}(k\bar{R}) \tag{5}$$

where we have introduced the tensor

$$\begin{aligned}\tau_{\ell m}(kR) &= \frac{1}{4\pi}\int d\Omega_k \left(\delta_{\ell m} - \hat{k}_\ell \hat{k}_m\right) e^{\pm i\mathbf{k}\cdot\mathbf{R}} \\ &= \left(-\nabla^2\delta_{\ell m} + \nabla_\ell\nabla_m\right)^R \frac{\sin kR}{k^3 R}\end{aligned} \tag{6}$$

with $R = \mid \mathbf{r}' - \mathbf{r} \mid$ and $\bar{R} = \mid \mathbf{r}' - \sigma\mathbf{r} \mid$. The matrix

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{7}$$

gives a reflection on the conducting plate, supposed orthogonal to the z axis. The second term of (5) is the effect of the infinite conducting plate on the field correlation function, which is thus modified with respect to the free-space case. Thus we expect a modification of the Casimir-Polder potential between two neutral atoms/molecules when the plate is present.

III. THE CASIMIR-POLDER POTENTIAL BETWEEN TWO ATOMS IN THE PRESENCE OF A CONDUCTING PLATE

We now consider the Casimir-Polder long-range interaction between the two atoms A and B, in the presence of the infinite perfectly conducting plate. We shall use two different approaches to calculate the potential, which also give new physical insights on the origin of the modification of the potential due to the plate. They also suggest possible extensions to more complicated boundary conditions. The methods we shall use have already been used for two- and three-body stationary Casimir-Polder potential [13, 14, 15], as well as for dynamical Casimir-Polder forces [16, 17], in the free space.

Using an appropriate transformation, the interaction terms in Hamiltonian (1) can be transformed to the following effective interaction Hamiltonian [14]

$$H = -\frac{1}{2}\sum_{\mathbf{k}j}\sum_{\mathbf{k}'j'}\alpha_A(k)\mathbf{D}(\mathbf{k}j,\mathbf{r}_A)\cdot\mathbf{D}(\mathbf{k}'j',\mathbf{r}_A) - \frac{1}{2}\sum_{\mathbf{k}j}\sum_{\mathbf{k}'j'}\alpha_B(k)\mathbf{D}(\mathbf{k}j,\mathbf{r}_B)\cdot\mathbf{D}(\mathbf{k}'j',\mathbf{r}_B) \tag{8}$$

where

$$\alpha(k) = \frac{2}{3\hbar c}\sum_p \frac{k_{p0}\mu_{p0}^2}{k_{p0}^2 - k^2} \tag{9}$$

is the atomic dynamical isotropic polarizability (for simplicity, we are assuming isotropic atoms) , $\hbar c k_{p0} = (E_p - E_0)$ is transition energy from atomic state p to the ground state 0 and μ_{p0} are matrix elements of the atomic dipole momentum operator.

In order to calculate the Casimir-Polder potential between the two atoms in the presence of the conducting plate, we first obtain the dressed ground state of one atom (A); then we evaluate the interaction energy of the other atom (B) with the field fluctations dressing atom A. Thus, the first step is to calculate the dressed ground state of one of the two atoms (let say A, with position $\mathbf{r}_A$), in the presence of the plate, and then to evaluate the average value

on this state of the effective interaction Hamiltonian of the second atom (B, with position $\mathbf{r}_B$) with the field.

The dressed ground state of atom A at the lowest significant order in the atom-radiation coupling, using straightforward perturbation theory, is

$$|\{0_{\mathbf{k}j}\},\downarrow_A\rangle_D = |\{0_{\mathbf{k}j}\},\downarrow_A\rangle - \frac{\pi}{V}\sum_{\mathbf{k}j}\sum_{\mathbf{k}'j'}\alpha_A(k)\frac{(kk')^{1/2}}{k+k'}\mathbf{f}(\mathbf{k}j;\mathbf{r}_A)\cdot\mathbf{f}(\mathbf{k}'j';\mathbf{r}_A)|1_{\mathbf{k}j}1_{\mathbf{k}'j'},\downarrow_A\rangle \quad (10)$$

The interaction energy with atom B is given by the average value of the effective interaction Hamiltonian of atom B with the field, evaluated on the dressed ground state of atom A, given by (10),

$$\begin{aligned}\Delta E_{AB} &= -\frac{1}{2}\sum_{\mathbf{k}j}\sum_{\mathbf{k}'j'}\alpha_B(k)\ {}_D\langle\{0_{\mathbf{k}j}\},\downarrow_A|\mathbf{D}(\mathbf{k}j,\mathbf{r}_B)\cdot\mathbf{D}(\mathbf{k}'j',\mathbf{r}_B)|\{0_{\mathbf{k}j}\},\downarrow_A\rangle_D \\ &= -\frac{2\pi^2\hbar c}{V^2}\sum_{\mathbf{k},\mathbf{k}'}(\alpha_A(k)+\alpha_A(k'))\alpha_B(k)\frac{kk'}{k+k'} \\ &\times \left[\sum_j f_\ell(\mathbf{k}j;\mathbf{r}_B)f_m(\mathbf{k}j;\mathbf{r}_A)\right]\left[\sum_{j'} f_\ell(\mathbf{k}'j';\mathbf{r}_B)f_m(\mathbf{k}'j';\mathbf{r}_A)\right] \end{aligned} \quad (11)$$

Using

$$\sum_j f_\ell(\mathbf{k}j;\mathbf{r})f_m(\mathbf{k}j;\mathbf{r}') = [\delta_{\ell m}-\hat{k}_\ell\hat{k}_m]e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} - \sigma_{\ell n}[\delta_{nm}-\hat{k}_n\hat{k}_m]e^{i\mathbf{k}\cdot(\mathbf{r}-\sigma\mathbf{r}')} \quad (12)$$

where σ is reflection matrix with respect to the plate (7), after substitution of (12) into (11) and in the continuous limit, we finally obtain

$$\begin{aligned}\Delta E_{AB} &= -\frac{\hbar c}{2(2\pi)^4}\int\int d^3k d^3k'(\alpha_A(k)+\alpha_A(k'))\alpha_B(k)\frac{kk'}{k+k'} \\ &\times \left([\delta_{\ell m}-\hat{k}_\ell\hat{k}_m]e^{i\mathbf{k}\cdot(\mathbf{r}_B-\mathbf{r}_A)} - \sigma_{\ell n}[\delta_{mn}-\hat{k}_m\hat{k}_n]e^{i\mathbf{k}\cdot(\mathbf{r}_B-\sigma\mathbf{r}_A)}\right) \\ &\times \left([\delta_{\ell m}-\hat{k}'_\ell\hat{k}'_m]e^{i\mathbf{k}'\cdot(\mathbf{r}_B-\mathbf{r}_A)} - \sigma_{\ell p}[\delta_{mp}-\hat{k}'_m\hat{k}'_p]e^{i\mathbf{k}'\cdot(\mathbf{r}_B-\sigma\mathbf{r}_A)}\right) \end{aligned} \quad (13)$$

This is the general expression of the Casimir-Polder potential energy between the two atoms in the presence of the conducting plate. In the so-called far zone, that is for interatomic distances larger than the significant atomic transition wavelengths from the ground state, we can approximate dynamical with static polarizabilities ($\alpha_{A,B}(k)\simeq\alpha_{A,B}(0)$), obtaining

$$\begin{aligned}\Delta E_{AB} &= -\frac{\hbar c}{(2\pi)^4}\alpha_A(0)\alpha_B(0)\int\int d^3k d^3k'\frac{kk'}{k+k'} \\ &\times \left([\delta_{\ell m}-\hat{k}_\ell\hat{k}_m]e^{i\mathbf{k}\cdot\mathbf{R}} - \sigma_{ln}[\delta_{mn}-\hat{k}_m\hat{k}_n]e^{i\mathbf{k}\cdot\bar{\mathbf{R}}}\right) \\ &\times \left([\delta_{\ell m}-\hat{k}'_\ell\hat{k}'_m]e^{i\mathbf{k}'\cdot\mathbf{R}} - \sigma_{\ell p}[\delta_{mp}-\hat{k}'_m\hat{k}'_p]e^{i\mathbf{k}'\cdot\bar{\mathbf{R}}}\right) \end{aligned} \quad (14)$$

where we have used $\mathbf{R} = \mathbf{r}_B - \mathbf{r}_A$ and $\bar{\mathbf{R}} = \mathbf{r}_B - \sigma \mathbf{r}_A$; $\bar{\mathbf{R}}$ is the distance between one atom and the image of the other atom with respect of the plate.

After algebraic calculations we obtain

$$\begin{aligned}
\Delta E_{AB}(R, \bar{R}) \;=\; & -\frac{23}{4\pi}\hbar c \frac{\alpha_A(0)\alpha_B(0)}{R^7} - \frac{23}{4\pi}\hbar c \frac{\alpha_A(0)\alpha_B(0)}{\bar{R}^7} \\
+\; & \frac{8}{\pi}\hbar c \frac{\alpha_A(0)\alpha_B(0)}{R^3 \bar{R}^3 (R+\bar{R})^5} \Big(R^4 \sin^2\vartheta + 5R^3\bar{R}\sin^2\vartheta \\
+\; & R^2\bar{R}^2(6 + \sin^2\vartheta + \sin^2\bar{\vartheta}) + 5R\bar{R}^3\sin^2\bar{\vartheta} + \bar{R}^4\sin^2\bar{\vartheta} \Big)
\end{aligned} \tag{15}$$

where ϑ and $\bar{\vartheta}$ are respectively the angles of $\mathbf{R}$ and $\bar{\mathbf{R}}$ with the normal to the plate. We note the presence of three terms: the usual R^{-7} Casimir-Polder potential between the two atoms (as in absence of the plate), the $\bar{R}^{-7}$ Casimir-Polder-like interaction between one atom with the reflected image of the second atom, and a term depending from both variables R and $\bar{R}$. This result has been already obtained by fourth order perturbation theory [10]. We have now obtained the same result with a different and simpler method, which stresses the role of dressed field fluctuations modified by the boundary conditions given by the presence of the conducting plate.

The potential (15) can be also obtained with a different method, based on the properties of the spatial correlations of vacuum fluctuations, which are modified by the conducting plate as discussed in the previous Section. The basic idea is that the vacuum fluctuations of the electromagnetic field induce instantaneous dipole moments on the two atoms, and that these induced dipoles are correlated because vacuum fluctuations are spatially correlated. The Casimir-Polder potential energy then arises from the (classical) interaction between the two correlated induced dipoles [13] . The arguments used in this physical model are essentially classical, except for the assumption of the real existence of vacuum fluctuations, which invokes quantum aspects of the electromagnetic field. This makes the model very interesting from a fundamental point of view, giving insights on the origin of Casimir-Polder forces and stressing the role of zero-point fluctuations. In our case, as discussed in the previous Section, the conducting plate changes the spatial correlations of vacuum fluctuations and a change of the Casimir-Polder potential is thus expected. We show that this physical model is indeed correct, but, compared to the free-space case, a modification of the classical interaction energy between the two dipoles is also necessary, in order to take into account the image atoms.

The relation between the induced dipole moments in the atoms, assumed isotropic for simplicity, and the field is

$$\mu_\ell(\mathbf{k}, j) = \alpha(k) D_\ell(\mathbf{k}j, \mathbf{r}) \tag{16}$$

The average interaction energy between the induced and correlated atomic dipoles is

$$\begin{aligned} V_{AB} &= \sum_{\mathbf{k}j} \mu_\ell^A(\mathbf{k}, j)\mu_m^B(\mathbf{k}, j) V_{\ell m}(k, R, \bar{R}) \\ &= \sum_{\mathbf{k}j} \alpha_A(k)\alpha_B(k) \langle D_\ell(\mathbf{k}j, \mathbf{r}_A) D_m(\mathbf{k}j, \mathbf{r}_B)\rangle V_{\ell m}(k, R, \bar{R}) \end{aligned} \tag{17}$$

where the average is taken on the ground state of the field; the presence of the spatial correlation function of the transverse displacement field should be noted. $V_{lm}(k, R, \bar{R})$ is a classical interaction energy between the induced dipole moments of the two atoms. In the case of atoms in the free space, this interaction energy is the classical interaction energy between dipoles oscillating at frequency ck [13]. When boundary conditions are present, this potential must be appropriately changed in order to take into account the interaction with the image atoms. We therefore take the following form of the classical interaction energy

$$\begin{aligned} V_{\ell m}(k, R, \bar{R}) &= V_{\ell m}(k, R) - \sigma_{\ell p} V_{pm}(k, \bar{R}) \\ &= (\nabla^2 \delta_{\ell m} - \nabla_\ell \nabla_m)^R \frac{\cos kR}{R} - \sigma_{\ell p} (\nabla^2 \delta_{pm} - \nabla_p \nabla_m)^{\bar{R}} \frac{\cos k\bar{R}}{\bar{R}} \end{aligned} \tag{18}$$

where the superscript of the differential operators above indicate the variable on which the operators act. The first terms is the same used for atoms in the free space. The second term arises from the presence of the conducting plate, and has the form of a atom-image interaction.

Using (4) the interaction energy (17) becomes

$$V_{AB} = \sum_{\mathbf{k}} \alpha_A(k)\alpha_B(k) \frac{2\pi\hbar ck}{V} \left(\sum_j f_\ell(\mathbf{k}j; \mathbf{r}_A) f_m(\mathbf{k}j; \mathbf{r}_B) \right) V_{\ell m}(k, R, \bar{R}) \tag{19}$$

In the continuous limit

$$\sum_{\mathbf{k}} \Longrightarrow \frac{V}{(2\pi)^3} \int d\Omega_k \int k^2 dk \tag{20}$$

and using (5,6), we obtain

$$V_{AB} = \frac{\hbar c}{\pi} \int_0^\infty dk\, k^3 \alpha_A(k)\alpha_B(k) \left(\tau_{\ell m}(kR) - \sigma_{\ell n}\tau_{nm}(k\bar{R})\right) \left(V_{\ell m}(k, R) - \sigma_{\ell p} V_{pm}(k, \bar{R})\right) \tag{21}$$

Taking into account the familiar expression of the Casimir-Polder interaction between two atoms in the free space [18]

$$\begin{aligned} V_{CP}(R) &= \frac{\hbar c}{\pi}\int_0^\infty dk\, k^3\alpha_A(k)\alpha_B(k)\tau_{\ell m}(kR)V_{\ell m}(k,R) \\ &= -\frac{\hbar c}{\pi R^2}\int_0^{+\infty} du\, u^4\alpha_A(iu)\alpha_B(iu)\left(1+\frac{2}{uR}+\frac{5}{u^2R^2}+\frac{6}{u^3R^3}+\frac{3}{u^4R^4}\right)e^{-2uR} \end{aligned} \quad (22)$$

and also that $\sigma_{\ell n}\tau_{nm}(k\bar{R})\sigma_{\ell p}V_{pm}(k,\bar{R}) = \tau_{\ell m}(k\bar{R})V_{\ell m}(k,\bar{R})$), we immediately see from (21) the presence of $V_{CP}(R)$ and $V_{CP}(\bar{R})$, plus two extra terms with both variables R and $\bar{R}$. $V_{CP}(R)$ and $V_{CP}(\bar{R})$ are respectively the interaction between the two atoms, as in the absence of the conducting plate, and the interaction between one atom and the image of the other atom. The other terms in (21) contain both coordinates R and $\bar{R}$, and their physical interpretation is not so evident. When (21) is approximated to the far zone, replacing the dynamical polarizabilities with the static ones, the same result of equation (15) is obtained.

This result shows how the method for the calculation of Casimir-Polder forces based on field correlations must be modified when boundary conditions are present. In fact, there are two elements that must be considered. First, the expectation value of the field correlation function changes in the presence of the boundary condition compared to the case of the unbounded space, as expressed by (4). Secondly, the classical interaction energy between the correlated induced dipoles must be changed according to (18), in order to take into account the image dipoles too. This also gives helpful indications for a generalization to more complicated boundary conditions, such as cavities or dielectric objects.

The two methods we have used to calculate the Casimir-Polder potential yield the same result. Both methods are based on the common idea that electromagnetic field fluctuations induce real effects in matter. The mathematical equivalence of the two methods can be proved formally, similarly to the case of two atoms in the free space [19]. In fact, after some algebraic manipulation eq. (11) can be expressed in the following form

$$\begin{aligned} \Delta E_{AB} &= -\frac{4\pi^2\hbar c}{V}\sum_{\mathbf{k}j} k f_\ell(\mathbf{k}j;\mathbf{r}_B)f_m(\mathbf{k}j;\mathbf{r}_A) \\ &\times \left[\alpha_A(k)\frac{1}{V}\sum_{\mathbf{k}'j'}\frac{k'^2}{k'^2-k^2}f_\ell(\mathbf{k}'j';\mathbf{r}_B)f_m(\mathbf{k}'j';\mathbf{r}_A)\alpha_B(k')+\right. \\ &+ \left.\frac{1}{V}\sum_{\mathbf{k}'j'}\frac{k'^2}{k'^2-k^2}f_\ell(\mathbf{k}'j';\mathbf{r}_B)f_m(\mathbf{k}'j';\mathbf{r}_A)\alpha_A(k')\alpha_B(k')\right] \end{aligned} \quad (23)$$

In the continuous limit, after evaluation of angular integrals, we obtain

$$\begin{aligned}\Delta E_{AB} &= \frac{2\pi\hbar c}{V}\sum_{\mathbf{k}j} k f_\ell(\mathbf{k}j;\mathbf{r}_B) f_m(\mathbf{k}j;\mathbf{r}_A) \\ &\times\ \alpha_A(k)\alpha_B(k)[(\nabla^2\delta_{\ell m}-\nabla_\ell\nabla_m)^R\frac{\cos kR}{R} - \sigma_{\ell p}(\nabla^2\delta_{pm}-\nabla_p\nabla_m)^{\bar{R}}\frac{\cos k\bar{R}}{\bar{R}}] \\ &= \frac{2\pi\hbar c}{V}\sum_{\mathbf{k}j} k f_\ell(\mathbf{k}j;\mathbf{r}_B) f_m(\mathbf{k}j;\mathbf{r}_A)\alpha_A(k)\alpha_B(k)[V_{\ell m}(k,R) - \sigma_{\ell p}V_{pm}(k,\bar{R})] \\ &= \frac{2\pi\hbar c}{V}\sum_{\mathbf{k}j}\alpha_A(k)\alpha_B(k)\langle 0_{\mathbf{k}j} \mid D_\ell(\mathbf{k}j,\mathbf{r}_A) D_m(\mathbf{k}j,\mathbf{r}_B) \mid 0_{\mathbf{k}j}\rangle V_{\ell m}(k,R,\bar{R}) \qquad (24)\end{aligned}$$

This relation indeed shows that the intuitive model based on spatial field correlations can be derived from the Hamiltonian (1), even when a boundary conditions such as an infinite conducting plate is present. This gives further support to this physical model, and we expect it should be valid also in the case of more complicated boundary conditions.

IV. CONCLUSIONS

In this paper we have first considered vacuum field fluctuations in the presence of a perfectly conducting plate, and discussed how they are modified compared with the free-space case. We have then considered the modification to the Casimir-Polder intermolecular potential between two atoms due to the presence of the plate, as a result of the modified zero-point fluctuations . We have calculated the Casimir-Polder potential using two different methods, one based on dressed vacuum fluctuations and the other on spatial vacuum field correlations. Our results agree with previous results obtained by fourth-order perturbation theory. Our methods have however two advantages. The first is that they are mathematically simpler, and this may be particularly relevant in more complicated situations; the second is that they give new physical insights on the role of boundary conditions on Casimir-Polder forces. Furthermore, the methods used in this paper give indications on possible generalizations to more complicated boundary conditions.

Acknowledgments

The authors wish to thank F. Persico for many discussions and suggestions on the subject of this paper. This work was in part supported by the bilateral Italian-Belgian project on

"Casimir-Polder forces, Casimir effect and their fluctuations" and by the bilateral Italian-Japanese project 15C1 on "Quantum Information and Computation" of the Italian Ministry for Foreign Affairs. Partial support by Ministero dell'Università e della Ricerca Scientifica e Tecnologica and by Comitato Regionale di Ricerche Nucleari e di Struttura della Materia is also acknowledged.

---

[1] H.B.G. Casimir, Phys. Rev. **73**, 360 (1948)

[2] M. Bordag, U. Mohideen, V.M. Mostepanenko, Phys. Rep. **353**, 1 (2001)

[3] A. Lambrecht, S. Reynaud, Sem. Poincaré **1**, 79 (2002)

[4] V. Druzhinina, M. DeKieviet, Phys. Rev. Lett. **91**, 193202 (2003)

[5] A. Aspect, J. Dalibard, Sem. Poincaré **1**, 67 (2002)

[6] H. Khosravi, R. Loudon, Proc. Roy. Soc. London A **433**, 337 (1991)

[7] K.A. Milton, *The Casimir Effect: Physical Manifestations of Zero-Point Energy*, World Scientific, Singapore 2001

[8] R. Weigand, J.M. Guerra, Eur. J. Phys. **18**, 40 (1997)

[9] M. Marcovitch, H Diamant, Phys. Rev. Lett. **95**, 223203 (2005)

[10] E.A. Power, T. Thirunamachandran, Phys. Rev. A **25**, 2473 (1982)

[11] G. Barton, Physica Scripta **T21**, 11 (1988)

[12] V Sopova, L.H. Ford, arXiv:quant-ph/0504143

[13] E.A. Power, T. Thirunamachandran, Phys. Rev. A **48**, 4761 (1993)

[14] R. Passante, E.A. Power, T. Thirunamachandran, Phys. Lett. A **249**, 77 (1998)

[15] M. Cirone, R. Passante, J. Phys. B: At. Mol. Opt. Phys. **30**, 5579 (1997)

[16] L. Rizzuto, R. Passante, F. Persico, Phys. Rev. A **70**, 012107 (2004)

[17] R. Passante, F. Persico, L. Rizzuto, Phys. Rev. Lett., submitted (2005)

[18] G. Compagno, R. Passante, F. Persico, *Atom-Field Interactions and Dressed Atoms*, Cambridge University Press, Cambridge 1995

[19] R. Passante, F. Persico, L. Rizzuto, Phys. Lett. A **316**, 29 (2003)